# LEARNING INVARIANT REPRESENTATIONS OF PLANAR CURVES

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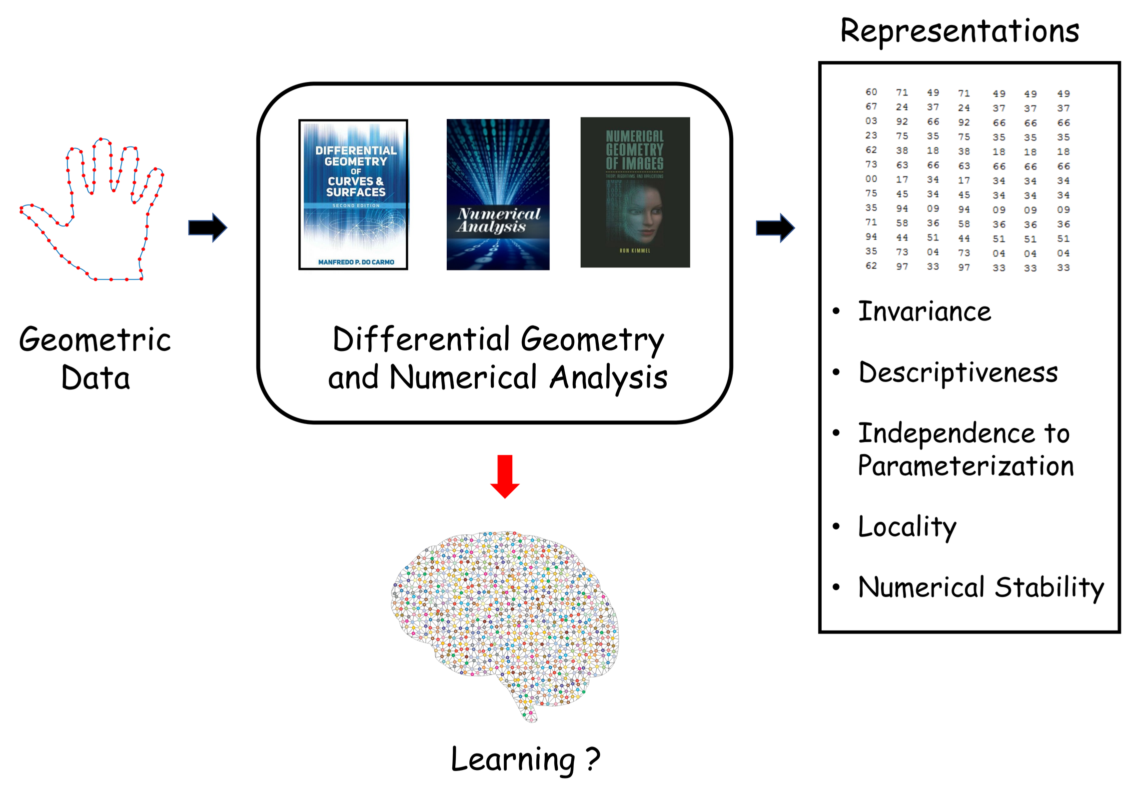
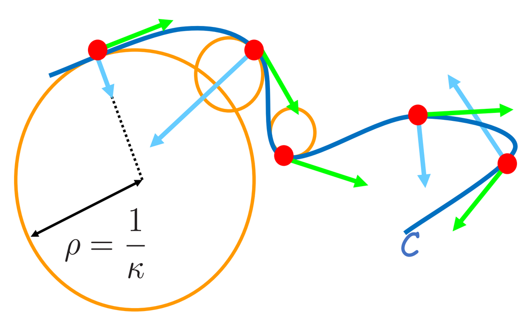
## MOTIVATION




## INVARIANT SIGNATURES OF PLANAR CURVES

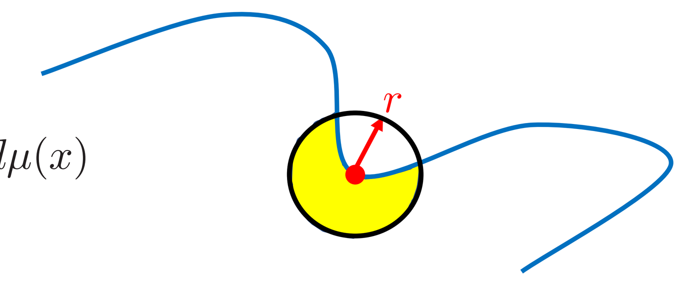
### Differential Invariants

$$\kappa(p) = \frac{x_p\, y_{pp} - y_p\, x_{pp}}{(x_p^2 + y_p^2)^{\frac{3}{2}}}$$




### Integral Invariants

$$I_r(p) = \int_{\mathcal{C}} h_r(p,x)\, d\mu(x)$$

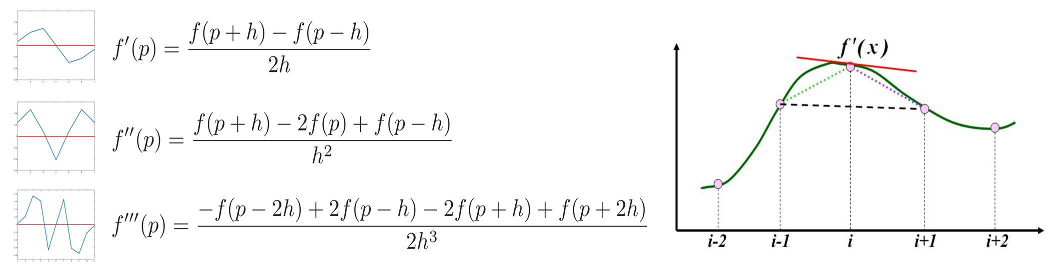



**Computational Action: Numerical Differentiation**

$$f'(p) = \frac{f(p+h) - f(p-h)}{2h}$$

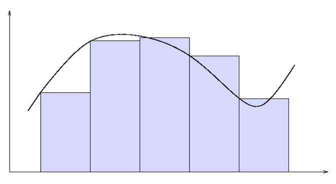
$$f''(p) = \frac{f(p+h) - 2f(p) + f(p-h)}{h^2}$$

$$f'''(p) = \frac{-f(p-2h) + 2f(p-h) - 2f(p+h) + f(p+2h)}{2h^3}$$



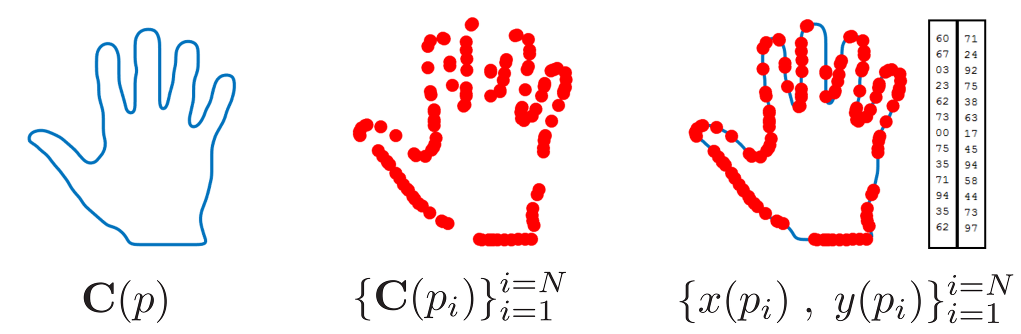

**Computational Action: Numerical Integration**

$$\int_a^b f(x)\, dx = \sum_{k=1}^{k=N} (x_{k+1} - x_k)\left(\frac{f(x_{k+1}) + f(x_k)}{2}\right)$$

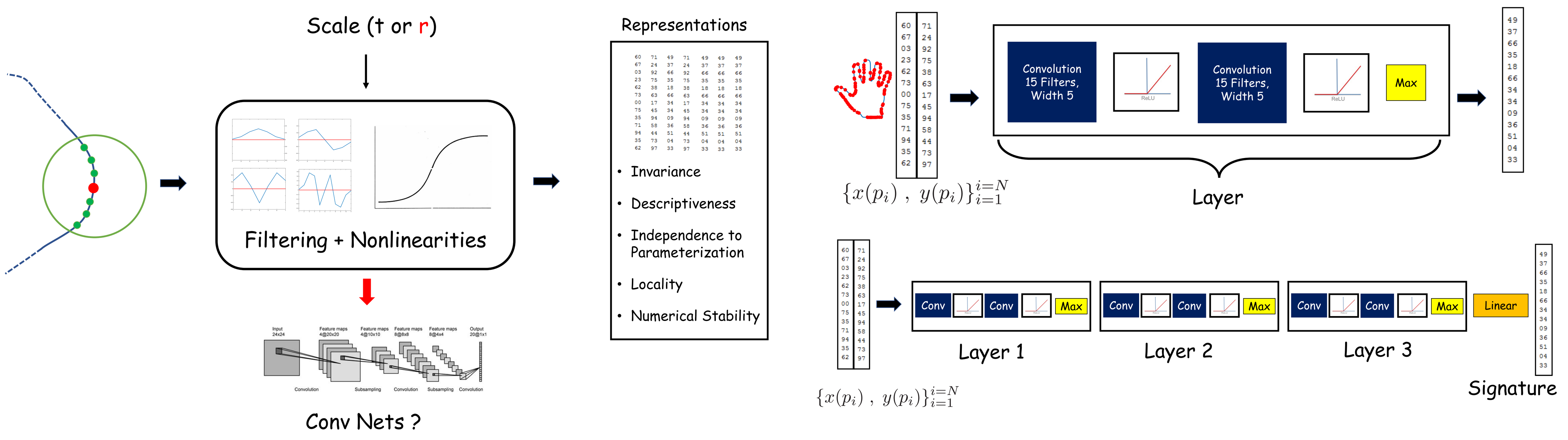


## CURVES

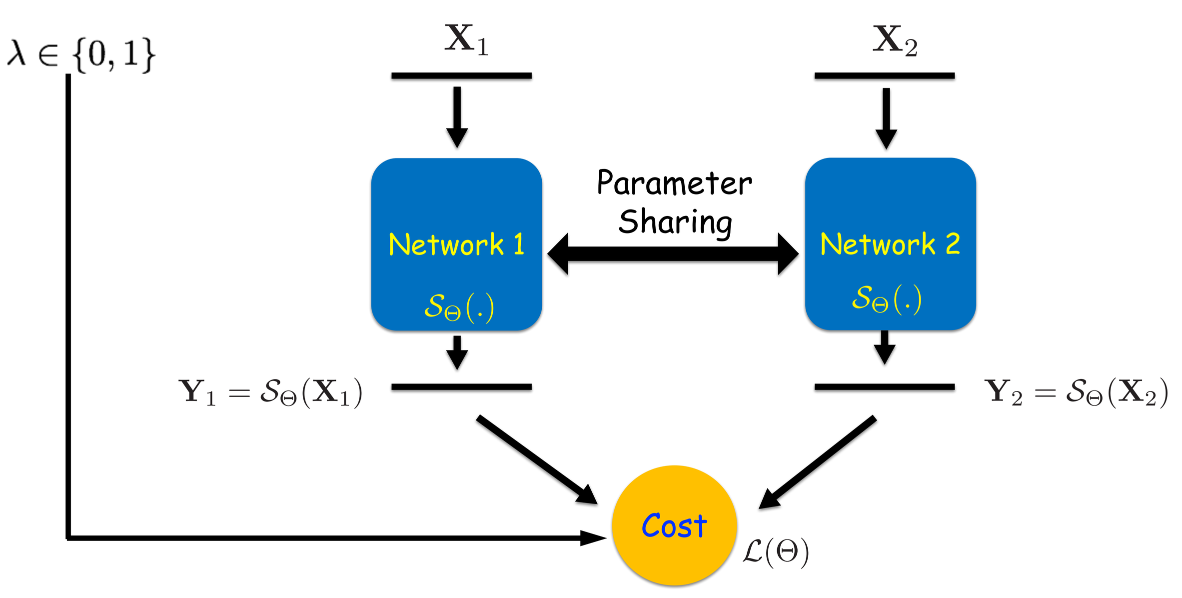
$$\mathbf{C}(p) = \{x(p),\ y(p)\}\quad p \in [0,1]$$




## COMPUTATIONAL OVERVIEW AND ARCHITECTURE

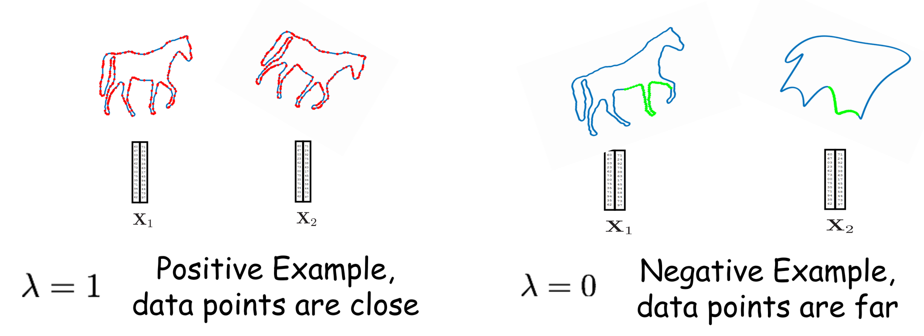



## SIAMESE CONFIGURATION



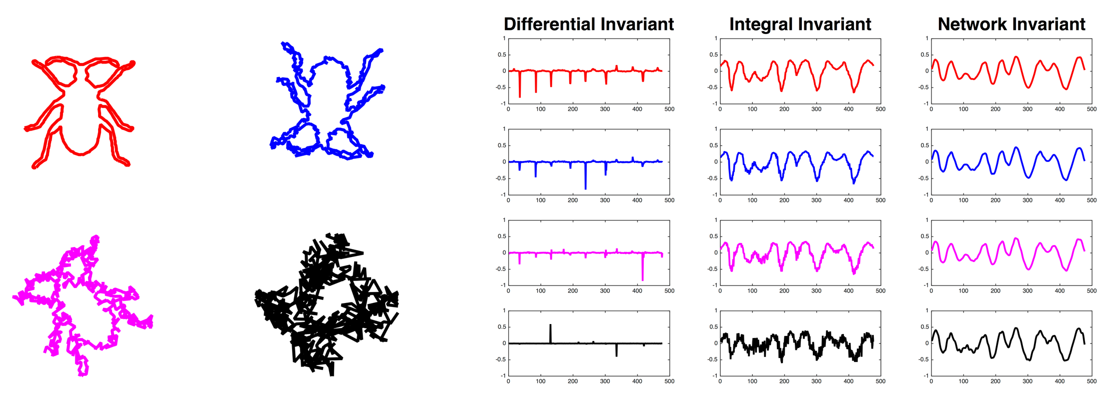

Minimize w.r.t. $\Theta$

$$\text{Cost} = \underbrace{\lambda\|\mathbf{Y}_1 - \mathbf{Y}_2\|}_{\text{Invariance}} + \underbrace{(1-\lambda)\max\{0, \mu - \|\mathbf{Y}_1 - \mathbf{Y}_2\|\}}_{\text{Descriptiveness}}$$

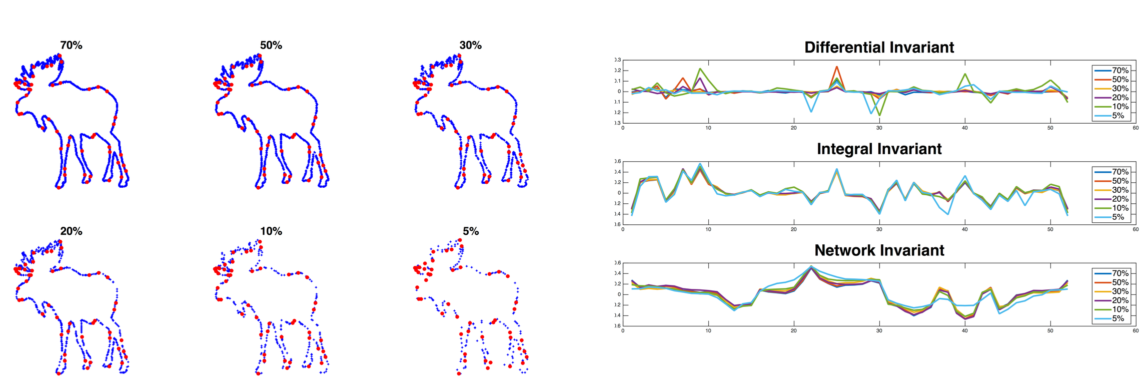



## NUMERICAL PROPERTIES

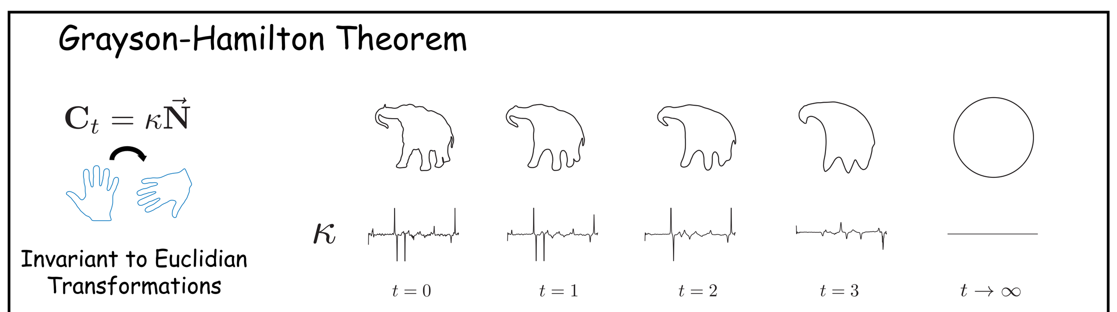
### Robustness To Noise

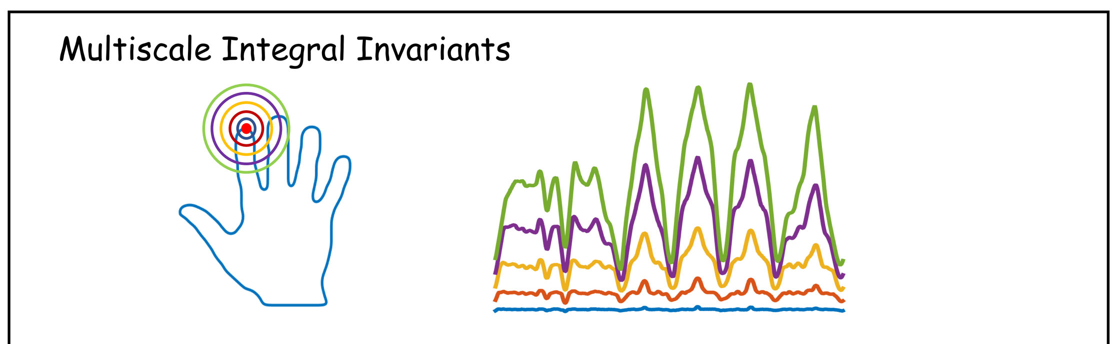


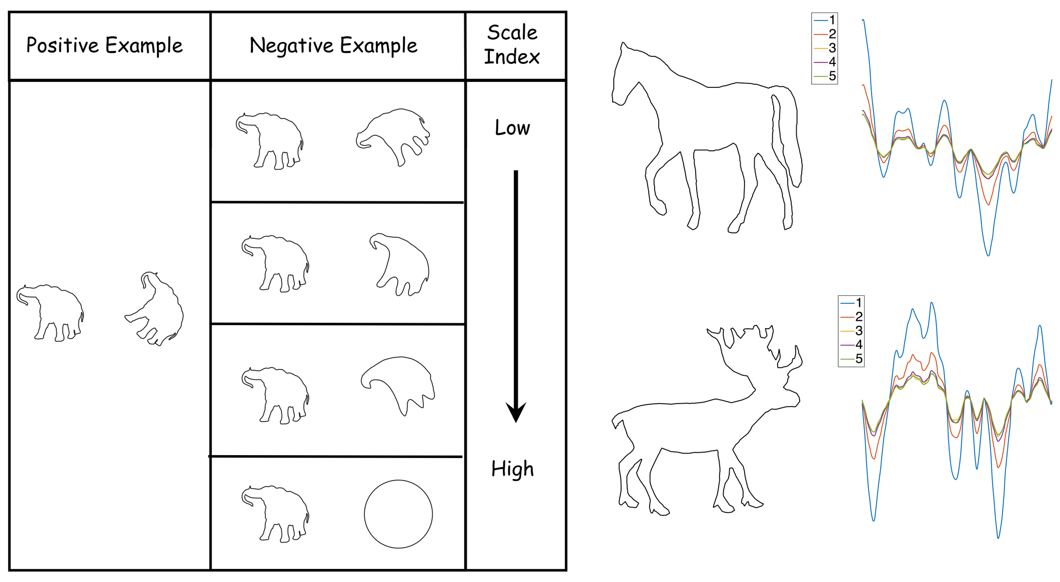

### Robustness To Sampling




## LEARNING MULTI-SCALE REPRESENTATIONS

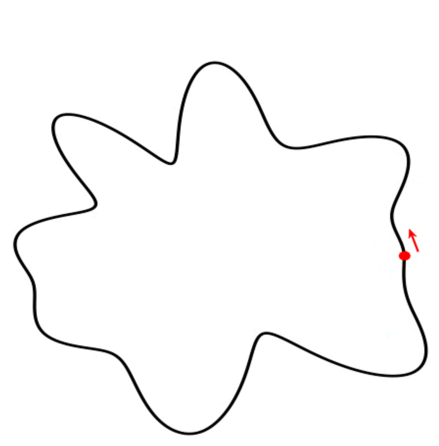


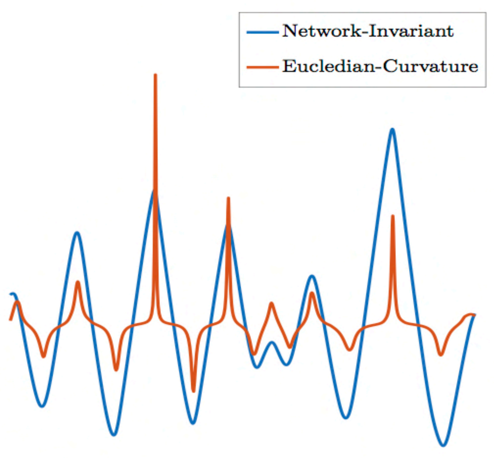




## INSIGHT AND VISUALIZATION

### Visual Comparison with Euclidian Curvature



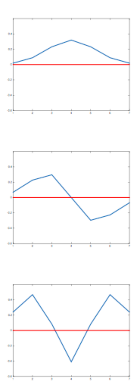



### Standard 1D Gaussian filters and its derivatives

$$g(x,\sigma) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}}$$

$$\frac{d}{dx}\, g(x,\sigma) = -\frac{x}{\sigma^3\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}}$$

$$\frac{d^2}{dx^2}\, g(x,\sigma) = -\frac{\sigma^2 - x^2}{\sigma^5\sqrt{2\pi}}\, e^{-\frac{x^2}{2\sigma^2}}$$



### Visualizing filters from the first layer