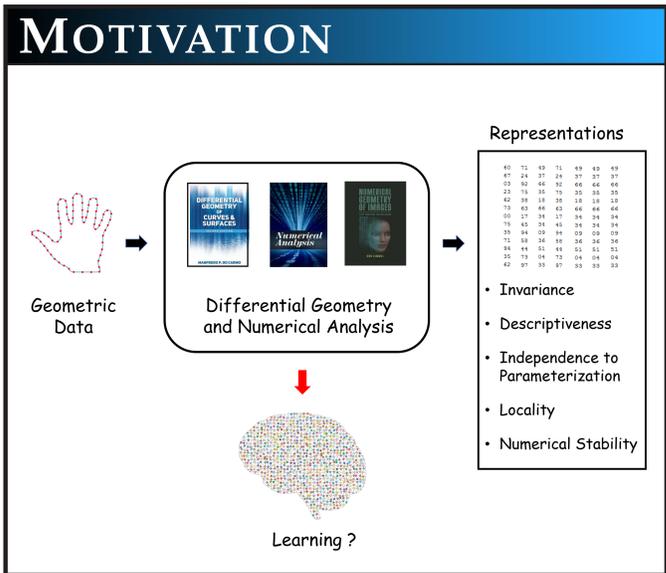


## MOTIVATION



**Geometric Data** → **Differential Geometry and Numerical Analysis** → **Representations**

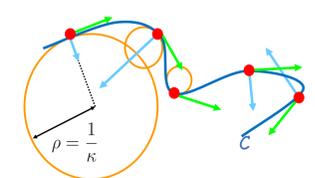
**Learning ?**

**Representations**

- Invariance
- Descriptiveness
- Independence to Parameterization
- Locality
- Numerical Stability

## INVARIANT SIGNATURES OF PLANAR CURVES

### Differential Invariants

$$\kappa(p) = \frac{x_p y_{ppp} - y_p x_{ppp}}{(x_p^2 + y_p^2)^{3/2}}$$


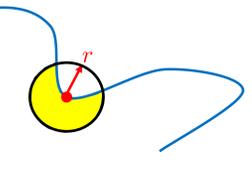
**Computational Action: Numerical Differentiation**

$$f'(p) = \frac{f(p+h) - f(p-h)}{2h}$$

$$f''(p) = \frac{f(p+h) - 2f(p) + f(p-h)}{h^2}$$

$$f'''(p) = \frac{-f(p-2h) + 2f(p-h) - 2f(p+h) + f(p+2h)}{2h^3}$$

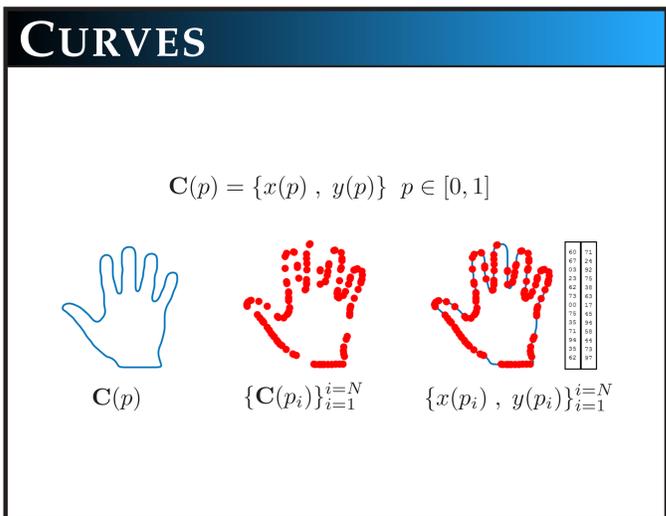
### Integral Invariants

$$I_r(p) = \int_C h_r(p, x) d\mu(x)$$


**Computational Action: Numerical Integration**

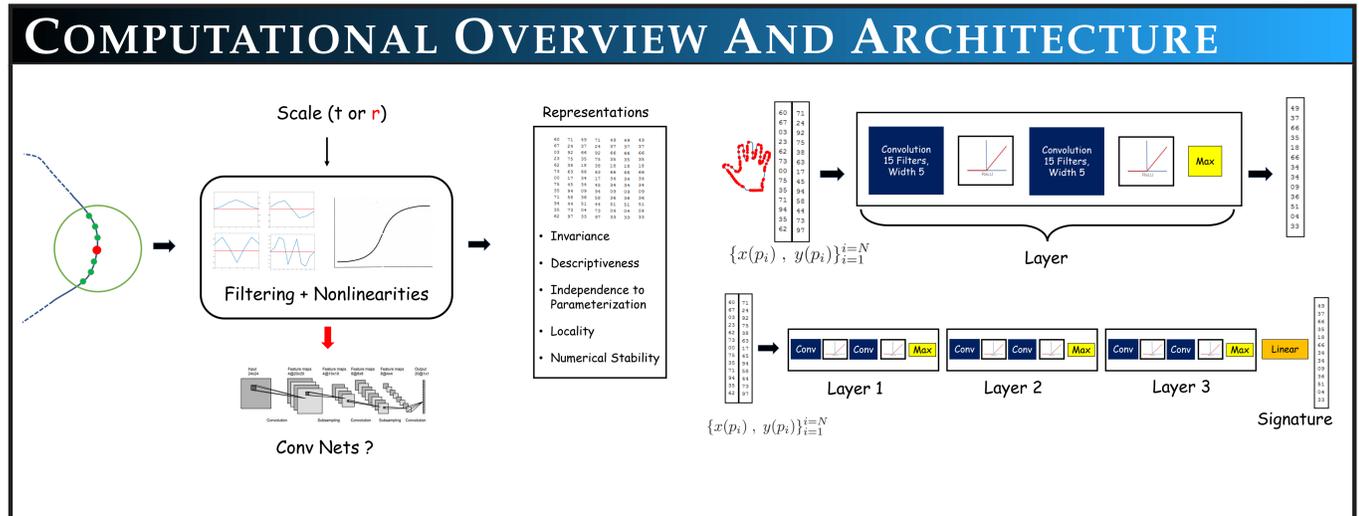
$$\int_a^b f(x) dx = \sum_{k=1}^{k=N} (x_{k+1} - x_k) \left( \frac{f(x_{k+1}) + f(x_k)}{2} \right)$$

## CURVES

$$C(p) = \{x(p), y(p)\} \quad p \in [0, 1]$$


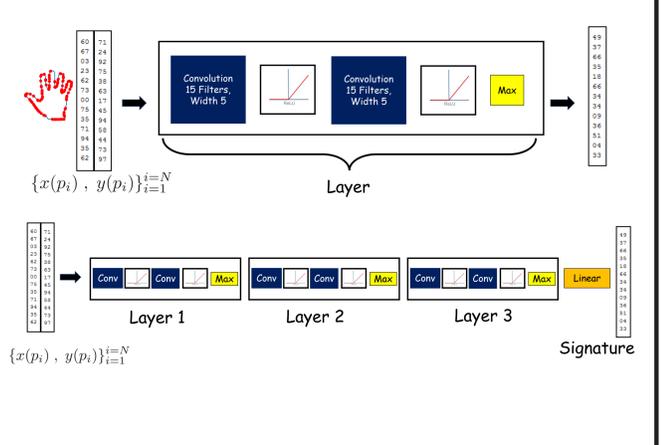
$\{C(p_i), y(p_i)\}_{i=1}^N$

## COMPUTATIONAL OVERVIEW AND ARCHITECTURE



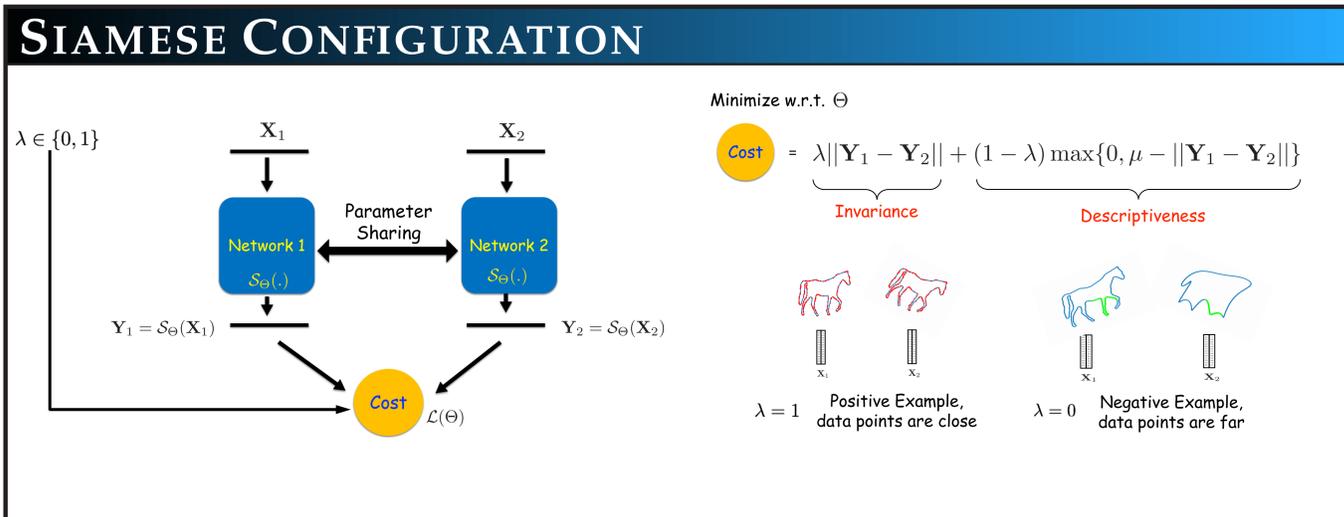
**Scale (t or r)** → **Filtering + Nonlinearities** → **Conv Nets ?** → **Representations**

- Invariance
- Descriptiveness
- Independence to Parameterization
- Locality
- Numerical Stability



**Layer 1** → **Layer 2** → **Layer 3** → **Linear** → **Signature**

## SIAMESE CONFIGURATION



$\lambda \in \{0, 1\}$

$Y_1 = S_\Theta(X_1)$     $Y_2 = S_\Theta(X_2)$

**Cost**  $\mathcal{L}(\Theta)$

Minimize w.r.t.  $\Theta$

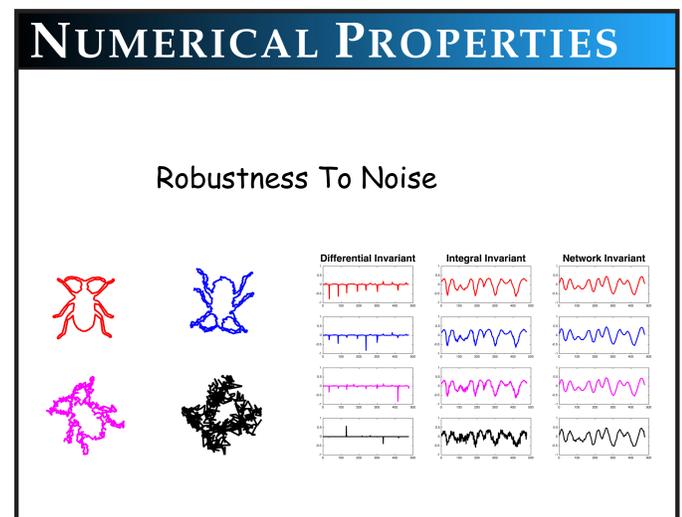
$$\text{Cost} = \lambda \|Y_1 - Y_2\| + (1 - \lambda) \max\{0, \mu - \|Y_1 - Y_2\|\}$$

**Invariance**   **Descriptiveness**

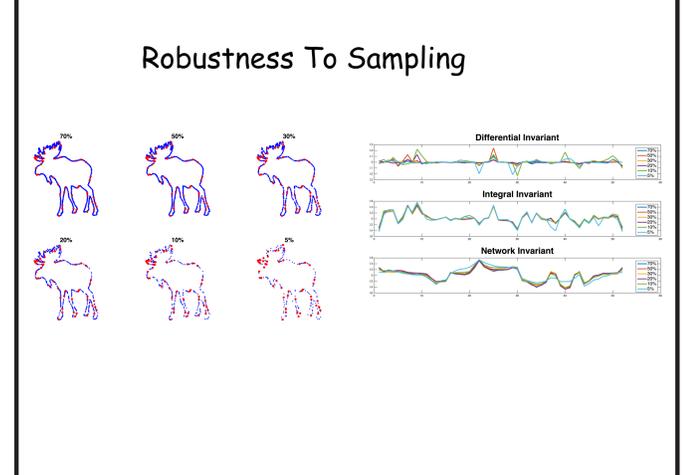
$\lambda = 1$  Positive Example, data points are close    $\lambda = 0$  Negative Example, data points are far

## NUMERICAL PROPERTIES

### Robustness To Noise



### Robustness To Sampling

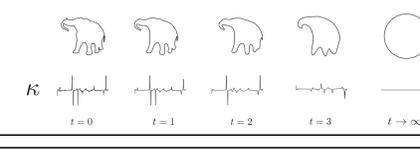


## LEARNING MULTI-SCALE REPRESENTATIONS

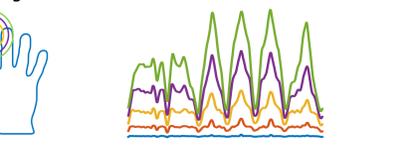
### Grayson-Hamilton Theorem

$$C_t = \kappa \vec{N}$$

Invariant to Euclidian Transformations

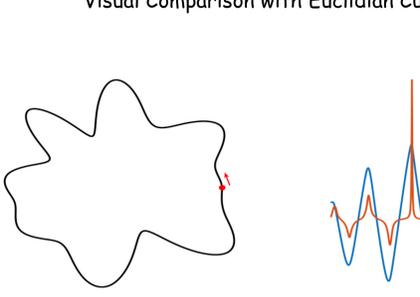


Positive Example	Negative Example	Scale Index
		Low
		↓
		High



## INSIGHT AND VISUALIZATION

### Visual Comparison with Euclidian Curvature



### Standard 1D Gaussian filters and its derivatives

$$g(x, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\frac{d}{dx} g(x, \sigma) = -\frac{x}{\sigma^3\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

$$\frac{d^2}{dx^2} g(x, \sigma) = \frac{x^2 - \sigma^2}{\sigma^5\sqrt{2\pi}} e^{-\frac{x^2}{2\sigma^2}}$$

### Visualizing filters from the first layer

